Product quality, advertising intensity and market size

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**Highlights**

- Consumer utility has a stochastic relationship with quality.
- Advertising intensity decreases with more firms.
- Product quality initially increases, then decreases with more firms.

**Abstract**

We develop a model of product differentiation in which firms strategically compete in product quality and advertising intensity. Products exhibit a combination of vertical and horizontal differentiation. A consumer’s utility has a stochastic relationship with quality, but they are more likely to prefer a higher quality good. Consumers face a trade-off between higher quality goods and price. Increased competition leads to less advertising, but may result in higher or lower quality products offered in the market.

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1. Introduction

In a market with differentiated products, firms select product attributes to maximize profit. Usually, firms also determine how much informative advertising to undertake to make consumers aware of their products. These decisions will depend, in part, on the number of competitors operating in the market.

The impacts of advertising depend on the type of product differentiation and the number of firms. With horizontal differentiation, consumers have heterogeneous preferences over quality. LeBlanc (1998) investigates informative price advertising in a horizontally differentiated, Hotelling duopoly model. He finds that informative advertising increases the firms’ expected profits. Hamilton (2009) examines a duopoly also and finds that informative advertising may be over or undersupplied depending on the degree of differentiation. Christou and Vettras (2008) show that firms in horizontally differentiated market may increase their profits by charging higher prices when using informative advertising, and increasing the number of firms in the market affects profits. Grossman and Shapiro (1984) suggest that advertising will decrease with the number of firms in the market.

Vertical differentiation occurs when consumers agree on a quality rank, but may value quality differently (Mussa and Rosen, 1978). Brouhle and Khanna (2007) suggest that providing more information about the quality characteristics of such products can increase the quality provided and may increase or decrease the welfare.

We consider a model that combines elements of horizontal and vertical differentiation. In our model, products vary by quality, and consumers’ utilities have a stochastic relationship between the product qualities. A quality rank exists, as with vertical differentiation. Consumers value quality differently as with horizontal differentiation. Specifically, consumers are more likely to prefer a higher quality good. We consider quality as a single attribute of the product that characterizes a variety. The objective of this work is to understand how given a quality differentiated market and consumers with heterogeneous preferences over quality, the number of firms...
in the market affects the amount of advertising and the quality of products produced.

We contribute to the literature by allowing consumer preferences to vary over quality in a non-uniform way. We analyze the existence of a symmetric equilibrium and the related comparative static properties, in order to identify the impacts of the number of firms in the market on the quality of goods provided and the advertising intensity.

2. Model

Consider a market with \( n \) firms (\( n \geq 2 \)) indexed by \( i \). Each firm sells a single product, which can be differentiated by quality. Consumers accurately become aware of the quality of the product once they receive a costly advertisement. Consumers are equally likely to receive any advertisement. We assume that advertising is not deceptive and consumers and firms take the quality and prices advertised as accurate.

Each firm has two choice variables—one is product quality \( q_i \); the other advertising intensity \( \phi_i \). Product quality represents the defining attribute of the product. How the quality varies across products reflects the degree of product differentiation in the market. Advertising intensity represents the percentage of consumers that receive an advertisement. Following Grossman and Shapiro (1984), we assume no fixed production cost and per unit production cost, \( C(q_i) \), increases with quality. Firms face an increasing advertising cost, \( A(\phi_i) \).

The population of consumers is normalized to 1, and each consumer has unit demand. A consumer \( \kappa \) who consumes one unit of the product of firm \( i \) receives value \( v_i^\kappa + u \). Consumers’ utility exhibits a stochastic relationship with the product’s characteristics, unlike the deterministic relationship in horizontal differentiation model of Grossman and Shapiro (1984) or vertical differentiation model in Mussa and Rosen (1978). The value of \( v_i^\kappa \), for any firm \( i \) and any consumer \( \kappa \), are randomly drawn from the distribution \( \Omega_i \), with density \( \omega \). We assume that \( \Omega_i \) represents a normal distribution, \( N(q, 1) \). Thus, \( F(v_i^\kappa) = q \). The expected utility of firm \( i \)’s product equals \( q_i + u \) and is increasing in quality. Therefore, consumers are more likely to prefer higher quality products. The reserve utility \( u \), associated with purchase from the class of heterogeneous goods, is the same across all consumers and assumed to equal zero.

Consumers purchase product \( i \) if they receive an advertisement from firm \( i \), and that product generates the highest net utility among the products of which they are aware. Here, \( Pr(v_i - p_i = \max(v_1 - p_1, v_2 - p_2, \ldots, v_k - p_k)) \) represents the probability that product \( i \) generates the highest utility as measured as value, \( v_i \), minus the price of the product, \( p_i \), for consumers of all \( k \) firms from whom consumers received advertisements.

We set \( q_i = q, p_i = p \) and \( \phi_i = \phi \) for firm \( j(\neq i) \) to derive the demand function of firm \( i \) as

\[
D(q, p, \phi) = \phi \sum_{k=0}^{n-1} \frac{(n-1-k)!}{k!} (1-\phi)^{n-k-1} \times \phi^k Pr(q, p, \phi, k),
\]

(1)

where \( Pr(q, p, \phi, k) = Pr(v_i - p_i > v - p)^k \). The demand for each firm will depend on the number of consumers that receive an advertisement, and the number of those consumers that select their product.

We write the profit function of firm \( i \) as

\[
\pi_i(q, p, \phi) = (p_i - C(q_i)) D(q_i, p_i, \phi) - A(\phi_i).
\]

(2)

3. Equilibrium

We begin by solving a symmetric equilibrium of identical firms similar to Christou and Vettas (2008). To solve the first-order conditions of firm \( i \)’s profit maximization problem, all firms other than firm \( i \) select the same quality, \( q \) and advertising intensity, \( \phi \), and face the same price, \( p \). We obtain,

\[
\frac{\partial \pi_i(q, q, p, \phi, \phi)}{\partial q_i} = -\frac{\partial C(q_i)}{\partial q_i} D_i(q_i, q, p, \phi, \phi) + (p_i - C(q_i)) \frac{\partial D_i(q_i, q, p, \phi, \phi)}{\partial q_i},
\]

(3)

where

\[
\frac{\partial D_i(q_i, q, p, \phi, \phi)}{\partial q_i} = \phi \sum_{k=0}^{n-1} \frac{(n-1)!}{(n-1-k)!} (1-\phi)^{n-k} \times \phi^k \frac{\partial Pr(q_i, q, p, \phi, k)}{\partial q_i}.
\]

In a symmetric equilibrium, firms maximize their profits at \( q_i = q \), which yields

\[
\frac{\partial D(q_i, q, p, \phi, \phi)}{\partial q_i} = \frac{1}{\sqrt{2\pi}} \phi^2 (n-1) \left(1 - \frac{1}{2}\phi\right)^{n-2}.
\]

(4)

The total market demand equals the probability that a consumer receives at least one advertisement, \( 1 - (1 - \phi)^n \). We write the per firm demand as

\[
D_i(q, q, p, p, \phi) = \frac{1}{n} \left(1 - (1 - \phi)^n\right).
\]

(5)

To obtain the first order condition with respect to the advertising intensity, we set all qualities equal to \( q \), price equal to \( p \) and all advertising intensities other than \( \phi_i \) equal to \( \phi \). The first order condition is

\[
\frac{\partial \pi_i(q, q, p, p, \phi, \phi)}{\partial \phi_i} = (p_i - C(q)) \frac{\partial D_i(q_i, q, p, \phi, \phi)}{\partial \phi_i} - \frac{\partial A(\phi_i)}{\partial \phi_i},
\]

(6)

and

\[
D_i(q, q, p, p, \phi, \phi) = \phi_i \sum_{k=0}^{n-1} \frac{(n-1)!}{(n-1-k)!} (1-\phi)^{n-k} \times \phi^k \frac{\partial Pr(q_i, q, p, \phi, k)}{\partial \phi_i}.
\]

(7)

Similarly, to generate the first order condition with respect to price, we set all qualities equal to \( q \), all advertising intensities equal to \( \phi \) and all prices other than \( p_i \) equal to \( p \). We obtain the first order condition as

\[
\frac{\partial \pi_i(q, q, p_i, p, \phi, \phi)}{\partial p_i} = (p_i - C(q)) \frac{\partial D_i(q_i, q, p_i, p, \phi, \phi)}{\partial p_i} + D_i(q, q, p_i, p, \phi, \phi),
\]

(8)

where

\[
\frac{\partial D_i(q_i, q, p_i, p, \phi, \phi)}{\partial p_i} = \phi_i \sum_{k=0}^{n-1} \frac{(n-1)!}{(n-1-k)!} (1-\phi)^{n-k} \times \phi^k \frac{\partial Pr(q_i, q, p_i, k)}{\partial p_i}.
\]

In a symmetric equilibrium, firms maximize their profits at \( p_i = p \), which yields

\[
\frac{\partial D_i(q_i, q, p, p, \phi, \phi)}{\partial p_i} = \frac{1}{\sqrt{2\pi}} \phi^2 (n-1) \left(1 - \frac{1}{2}\phi\right)^{n-2}.
\]

(9)

To analytically solve the symmetric equilibrium, we assume specific functional forms consistent with the model, a standard
Define the proof.

As thenumber of firms in the market increases, each firm advertises less.

When the number of firms in the market increases, each firm would increase the quality over the symmetric equilibrium by decreasing quality as the associated reduction in cost outweighs the loss in revenues. With a very large number of firms, firms would increase the quality over the symmetric equilibrium quality. In this case, the equilibrium quality is very low so the additional revenues associated with a higher quality would more than cover the small increase in costs. We provide a proof in Appendix A.

Proposition 1. As the number of firms in the market increases, each firm advertises less.

Proof. Define $L = \frac{(1-(1-\phi)^n)^2}{\phi \sqrt{2\pi} \sigma^2 (n-1) (1-1/2)^{n/2}} = 2\phi$. Using a first order Taylor expansion, $L \approx \frac{1}{\sqrt{2\pi} \sigma^2 (n-1) (1-1/2)^{n/2}}$. We can approximately obtain $\frac{dL}{dn} = -\frac{\partial L}{\partial n} = -\frac{1}{\sqrt{2\pi} \sigma^2 (n-1) (1-1/2)^{n/2}} < 0$, where $\partial L/\partial n = -\frac{1}{\sqrt{2\pi} \sigma^2 (n-1) (1-1/2)^{n/2}}$.

With few firms in the market the level of advertising is relatively high. As more firms enter the market, firms decide to reach fewer consumers with advertising as the cost savings of advertising less outweigh the expected increase in revenues from sustaining higher levels of advertising.

To determine the symmetric equilibrium price and quality, we need to define a relationship between the price and output when product differentiation exists. Based on Naimzada and Tramontana (2012), we assume a simple linear relationship,

$$p_i = a - D_i - \sum_{j=1, j\neq i}^n r_j D_j.$$  \hspace{1cm} (12)

Here, $p_i$ is the price of good $i$, $a$ is a parameter such that $a > 0$. $D$ is subscripted for each firm and represents each firm’s demand, and $r_j$ measures the degree of product differentiation or how the quality of products differ. When $r_j = 1$ the products are homogeneous, and when $r_j = 0$ the products are perfectly differentiated. In the symmetric equilibrium, firms provide a homogeneous product with the same quality. Therefore, we obtain the equilibrium market price as

$$p = a - nD.$$  \hspace{1cm} (13)

Proposition 2. The symmetric equilibrium quality increases and then decreases as the market size increases.

We can define the symmetric equilibrium quality from the expression (11).

$$q = p - (p - q) = a - nD - \frac{2\phi^2 n}{1 - (1 - \phi)^n}.$$  \hspace{1cm} (14)

(See Appendix B for the formal proof.)

Consumers generally prefer a higher quality and a lower price. Initially with relatively few firms in the market, firms will increase the product quality which will result in a higher price. The additional revenue will cover the increased production costs. As more firms enter the market, firms will begin to reduce the product quality. With more firms competing, each firm generates higher profit with lower quality and production costs and more units sold at a lower price.

In Fig. 1, with relatively few firms in the market, firms increase quality as the associated increase in price and resulting revenues outweigh the increased costs of production. However, as the number of firms approaches 10, the increased cost of production outweighs revenues and firms generate higher profit by reducing quality.

4. Conclusion

Our model represents a straightforward way to study the relationship between the number of firms in a market, product quality and informative advertising. Firms compete strategically in product quality and advertising. Consumer utility has a stochastic relationship with product quality, but consumers tend to prefer higher quality. Therefore, consumers face a tradeoff between price and quality. Advertising decreases with more firms. As more firms enter the market, the product quality initially increases, then decreases.

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Appendix A. Existence of the symmetric equilibrium

First, we consider firm i maintains the same product quality q as the other firms in the symmetric equilibrium, but changes advertising intensity by ∆φ to φ. We write firm i’s price as $p_i = a - D_i$ and the other firms’ price as $p = a - (n - 1)D$. Firm i chooses the defection strategy of a different advertising intensity if and only if the associated profit outweighs the profit under the symmetric equilibrium, $[p_i - C(q)]D(q, p_i, φ_i) - A(φ_i) > [p - C(q)]D(q, p, φ) - A(φ)$. The numerical results indicate that firms defect from the symmetric equilibrium advertising intensity when $n = 2$ with $\Delta φ \in [−0.14, −0.01]$ and $n \in [10, 30]$ with $\Delta φ_i \in [0.06, 0.20]$.

Next, firm i maintains the symmetric equilibrium advertising intensity φ, but changes the product quality by ∆q to q. Again, we begin with the model $p_i = a - D_i - \sum_{j=1,j\neq i}^n r_jD_j$. However, now we allow the defecting firm to change quality so we specifically define $r_j = e^{−|q_i|}$. Notice, $r_j \in [0, 1]$ and depends on the variation between product qualities. Firm i selects a product quality different from the symmetric equilibrium level if and only if given the following condition, $[p_i - C(q_i)]D(q_i, p_i, φ_i) - A(φ_i) > [p - C(q)]D(q, p, φ) - A(φ)$. The numerical results indicate that firms decrease quality from the symmetric equilibrium level when $n \in [15, 18]$ and increase quality from the symmetric equilibrium level when $n > 18$.

Combining the above analysis, the symmetric equilibrium only occurs when $n \in [3, 10]$.

Appendix B. Relationship of product quality and market size

We show that the first order derivative of product quality with respect to market size is positive when the market size is small and becomes negative when the market size is large.

From the expression (11), we can obtain

$$\frac{dq}{dn} = \frac{dp}{dn} \frac{dp - q}{dn}.$$  

First, we have

$$\frac{dp}{dn} = (1 - \phi)^n \ln(1 - \phi) - n(1 - \phi)^{n-1} \frac{d\phi}{dn}.$$  

where

$$\lim_{n \to 2} \frac{dp}{dn} = (1 - \phi)^n \ln(1 - \phi) + n(1 - \phi)^{n-1} > 0 \quad \text{and} \quad \lim_{n \to \infty} \frac{dp}{dn} = (1 - \phi)^n \ln(1 - \phi) < 0.$$

Second, we obtain

$$\frac{\partial(p - q)}{\partial n} = \frac{(2\phi^2 + 2n\phi \frac{dn}{dn})(1 - (1 - \phi)^n) - 2\phi^2 n(\frac{dn}{dn})(1 - \phi)^n \ln(1 - \phi))}{(1 - (1 - \phi)^n)^2},$$  

where

$$\lim_{n \to 2} \frac{\partial(p - q)}{\partial n} = \frac{2\phi(\phi - n)(1 - (1 - \phi)^n) + 2\phi^2 n(1 - \phi)^n \ln(1 - \phi)}{(1 - (1 - \phi)^n)^2} < 0$$  

and

$$\lim_{n \to \infty} \frac{\partial(p - q)}{\partial n} = \frac{2\phi^2}{1 - (1 - \phi)^n} > 0.$$

Combining the above results, we prove that $\frac{dp}{dn}$ is positive when $n$ goes to 2 and is negative when $n$ goes to infinity. It is implicit to conclude the product quality q is increasing with market size n and then decreasing with market size n.

References


